**Particle Swarm Optimization Algorithm**

Particle Swarm Optimization, also known as PSO, is a population-based metaheuristic algorithm that was proposed by Kennedy & Eberhart, (1995). The observation of social behaviors in creatures, such as fish schooling and bird flocking, inspired researchers to investigate how the cooperation of different species may help them achieve their objectives as a collective. In particle swarm optimization, each candidate for a solution is referred to as a "particle," and it navigates the search space for the problem like a bird swarm does when it is seeking for food, looking for the best possible place to settle (Salman et al., 2002). A particle's status on the search space can be identified by looking at two different vectors: its position vector and its velocity vector where is the number of dimensions of the solutions. In addition to this, the PSO algorithm has a memory, which means that every particle remembers the best position vector that it reached during the previous iterations and the best position that was reached by the best particle in the swarm, which is referred to as .

Similar to evolutionary algorithms, in PSO, each particle adjusts its velocity and position over each dimension in each iteration using the following equations (1) and (2):

|  |  |  |
| --- | --- | --- |
|  |  | () |

And,

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | () |

where represents the weight of the object's moment of inertia, and represent the acceleration coefficients, and and are two random numbers that fall within the range [0, 1].

However, the basic PSO that Kennedy and Eberhart proposed can only be utilized in the context of a constant m-dimensional continuous optimization problem. In contrast, the cut order planning problem is a discrete optimization problem with variables that must be integers and where number of variables (dimension) are not constant. Each potential solution of a COP problem may have a different number of sections. In fact, one of the outputs of the COP problem is the number of sections required to optimize the cost. Moreover, in the context of COP, updating the solution particle throughout each iteration is a very complicated process for a number of different reasons. It must generate solutions that are viable. It may produce solutions that do not necessarily have an equal number of sections. Therefore, in this research, we utilized a modified version of the combinatorial PSO (CPSO) that was proposed by Jarboui et al., (2008). The original CPSO suggested by Jarboui et al., (2008) also takes into account constant dimension for every particle solution but in a discrete combinatorial search space. Hence, we had to adapt it in order to apply it in our particular optimization problem. Here is how we designed discrete PSO for our COP problem:

Two dummy variables called & are employed to enable the change from the continuous state to the discrete state, and vice versa. and stand for the n-dimensional vector connected to the solution , where has discrete values between depending on the particle's state of solution at iteration .

Here, is the total number of sections in particle and is the minimum number of sections in . both are variables across the swarm.

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Now, represent the distance between the current solution, , and the best solution found by the particle, . And represent the distance between and the best solution found by the swarm , after iteration.

The velocity term update equation in this discrete PSO is then:

|  |  |
| --- | --- |
|  | (4) |

The velocity is then used in the Equation (5) to determine :

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Equation (6) is then used to modify the position vector of the particle where is the number of sections (dimensions) of the updated solution.

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

In this case, the parameter is the one that controls both the convergence and the divergence of the process. When has a relatively low value, it is likely that will take on either the value or the value (intensification). In the other scenario, we force the algorithm to pick a value that is distinct from both P and G (diversification).

Finally, the dimension of the velocity vector for the particle is updated to be equal to the dimension of the revised position vector , according to equation (7).

|  |  |  |
| --- | --- | --- |
|  |  | () |

**Pseudocode P: Discrete PSO Algorithm**

1. Generate Initial population that has ‘swarmsize’ amount of feasible solutions (particles) by applying H1,H3, & H5 and SET them to variable *‘Swarm’* where each particle is *XP*
2. SET the appropriate parameter values
3. For in swarmsize:
   1. SET *PbestP = XP*
4. Generate Initial velocity *VP*in range [a,b] for each particle *XP*
5. SET *Gbest* = Best position vector of a particle in the swarm with lowest fitness value (apply local search method)
6. FOR in iteration:
   1. FOR in swarmsize:
      1. Calculate by the Equation (3)
      2. Update velocity by the Equation (4)
      3. Calculate by the Equation (5)
      4. Update by calling function
      5. Update dimension of velocity vector by using Equation (7)
      6. IF Fitness () < Fitness ():
         1. SET =
      7. IF Fitness () < Fitness ():
         1. SET
7. RETURN

FUNCTION :

1. DETERMINE minimum number of sections in
2. INITIALIZE with 0 section.
3. : UPDATE residual demand for solution
4. FOR ):
   1. IF :
      1. : Copy section of (both ) and CREATE a new section for
   2. ELSE IF :
      1. Copy section of (both ) and CREATE a new section for
   3. ELSE:
      1. : Copy section of (both ) and CREATE a new section for
   4. : UPDATE residual demand for solution
   5. IF :
      1. STOP and RETURN
5. SET Total number of sections in
6. FOR :
   1. : Copy section of (both ) and CREATE a new section for
   2. : UPDATE residual demand for solution
   3. IF :
      1. STOP and RETURN
7. IF :
   1. Randomly USE or algorithm to pack all remain demands
8. RETURN

FUNCTION :

1. SET
2. IF Random Uniform [0,1] < *perturbation rate:*
   1. =
   2. SET = Random Integer
   3. IF , then SET
3. FOR :
   1. IF Random Uniform [0,1] < *perturbation rate:*
      1. SET
      2. DETERMINE length of the section
      3. DETERMINE
      4. SET
4. RETURN

Example

Let’s consider an order with the following demands:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Size1 | Size2 | Size3 | Size4 | Size5 |  |  |  |
|  | 7 | 23 | 26 | 17 | 13 |  |  |  |

Now after iteration, let’s consider a random particle solution, , it’s velocity vector , it’s personal best solution, and the Global best solution found in swarm :

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  | |  | |  | |
|  |  | | | | | |  | |  | |  |
|  | 0 | 0 | 0 | 1 | 1 | 9 | |  | | -0.48712 | |
| 0 | 1 | 1 | 0 | 0 | 22 | |  | | -0.34965 | |
| 1 | 0 | 1 | 0 | 0 | 5 | |  | | -0.09995 | |
| 0 | 0 | 0 | 1 | 0 | 7 | |  | | 0.37745 | |
| 0 | 1 | 0 | 0 | 1 | 4 | |  | | 0.27231 | |
| 0 | 0 | 0 | 1 | 0 | 4 | |  | | 0.22360 | |
| 1 | 0 | 0 | 0 | 0 | 4 | |  | | 0.30020 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | | |  | |  | |  | |
|  |  | | | | | |  | |  | |  | |
|  | 0 | 6 | 0 | 4 | 0 | 4 | |  | |  | |
| 2 | 0 | 3 | 1 | 4 | 4 | |  | |  | |
| 0 | 0 | 4 | 0 | 0 | 4 | |  | |  | |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | | |  |  | |  | |
|  |  | | | | | |  |  | |  | |
|  | 1 | 3 | 3 | 2 | 1 | 7 | |  |  | |
| 0 | 1 | 1 | 1 | 1 | 4 | |  |  | |
| 0 | 0 | 1 | 0 | 1 | 4 | |  |  | |

Initially, the value for the subsequent iteration is determined using Equation (3).  contains three values due to the fact that only the first three section of can be compared with and .

|  |  |
| --- | --- |
|  |  |
|  | 0 |
| 0 |
| 0 |

Next, the velocity of the particle at the iteration is determined by using Equation (4). Because there are only three possible values for , the velocity does not change after the third section.

|  |  |
| --- | --- |
|  |  |
|  | 0.68173 |
| 0.61888 |
| 0.11383 |
| 0.37745 |
| 0.27231 |
| 0.22360 |
| 0.30020 |

Now, from Equation (5) we get the .

|  |  |
| --- | --- |
|  |  |
|  | 0.68173 |
|  | 0.61888 |
|  | 0.11383 |

Following that, the particle solution will be revised using the function, which is primarily based on Equation (6). First, a new solution, , is formed with 0 section; then, the first value of is reviewed.

In our example, let's assume that . Given that , a complete section from is copied and added to the new particle solution . Considering that, the residual demands are then measured. While the demands are positive, we go to the subsequent value of . Now that , another section is therefore taken from and added to . At this point, the appearance of the updated particle is as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | |  |
|  |  | | | | |  |
|  | 1 | 3 | 3 | 2 | 1 | 7 |
|  | 0 | 1 | 1 | 1 | 1 | 4 |

And the current remaining demands are as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| Size1 | Size2 | Size3 | Size4 | Size5 |
| 0 | -2 | 1 | -1 | 2 |

As long as the numbers are still positive, we shall move on to the following value. At this point, . In accordance with Equation (6), the third section of is taken and temporarily added to .

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 0 | 1 | 0 | 0 | 5 |

After which, is subjected to a small perturbation. Initial consideration is given to perturbing the value. A uniform random number in the range [0,1] is created and compared to the given perturbation rate (). Suppose that the produced random number is less than . The value of will therefore be altered. To accomplish this, two integers and are determined as follows:

=

In our case Thus, we have , respectively. Consequently, we set = a random integer between [4,5] = 4.

The next step is perturbing the values. Assume that the value of Size1 is picked at random to be altered. To begin, we put . Then a temporary variable is obtained.

In our scenario, residual demand for Size l, , resulting in . Therefore, is updated to be . Following the perturbation, the section is permanently placed to and the residual demands are updated.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | |  |
|  |  | | | | |  |
|  | 1 | 3 | 3 | 2 | 1 | 7 |
|  | 0 | 1 | 1 | 1 | 1 | 4 |
|  | 0 | 0 | 1 | 0 | 0 | 4 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| Size1 | Size2 | Size3 | Size4 | Size5 |
| 0 | -2 | -3 | -1 | 2 |

Since there is no value left but the residual demand exist, we continue to add perturbated sections from until there are no residual demands or no section left in If there are any positive residual demands after adding all of the sections from , the H1 or H3 algorithms are chosen at random to pack all of the remaining demand to make the solution feasible. However, in our case, solution was completed after the addition of 2 more sections from.

Finally, the dimension of the velocity vector is adjusted to match the dimension of position vector . The new solution has only five sections, as opposed to the previous one, which had seven. Therefore, we retain the first five values of and eliminate the remainder.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | |  |  |  |
|  |  | | | | |  |  |  |
|  | 1 | 3 | 3 | 2 | 1 | 7 |  | 0.68173 |
|  | 0 | 1 | 1 | 1 | 1 | 4 |  | 0.61888 |
|  | 0 | 0 | 1 | 0 | 0 | 4 |  | 0.11383 |
|  | 0 | 0 | 0 | 1 | 0 | 6 |  | 0.37745 |
|  | 0 | 0 | 0 | 0 | 1 | 4 |  | 0.27231 |

The redesigned discrete PSO was applied to four distinct datasets, and the resulting graph for iteration versus fitness of global best solution looks somewhat like this.

Graphical user interface, histogram

Description automatically generated with medium confidenceA picture containing chart

Description automatically generatedChart, line chart

Description automatically generated